Computer Modeling Laboratory 8

Written report due: Nov.12

Solar radiative transfer approximations: single scattering approximation, Eddington and Delta-Eddington approximations

TASK 1

A. *Using the first order scattering approximation (Eq.[17.11]), show that the albedo of an optically thin layer is proportional to a product of single scattering albedo, phase function and optical depth of this layer.*

$$I_\lambda^\uparrow(0,\mu,\varphi)=\frac{\omega_0\mu_0F_0}{4\pi(\mu+\mu_0)}P(\mu,\varphi,-\mu_0,\varphi_o)\left[1-\exp\left(-\tau^*(\frac{1}{\mu}+\frac{1}{\mu_0})\right)\right]$$

In the assumption of thin atmosphere the optical depth, τ, is very sm2all, and exp(-τ)=1- τ, thus

$$I_\lambda^\uparrow(0,\mu,\varphi)=\frac{\omega_0\mu_0F_0}{4\pi(\mu+\mu_0)}P(\mu,\varphi,-\mu_0,\varphi_o)\left[\tau^*(\frac{\mu+\mu_0}{\mu\mu_0})\right]$$

Therefore, albedo of an optically thin layer is proportional to a product of single scattering albedo, phase function and optical depth of this layer:

$$R=\frac{\pi I_\lambda^\uparrow(0,\mu,\varphi)}{\mu_0F_0}=\frac{\omega_0\tau^*}{4\mu_0\mu}P$$

B. *In the above approximation, calculate and plot the upwelling radiance of the layer as a function of zenith angle for two sun zenith angles of 0^0 and 60^0. Take optical depth 0.1, single scattering albedo 0.95 and Henyey-Greenstein phase function with g=0.7 and solar flux F_0=1. Explain the differences between the radiances for the two sun angles.*

The Heney-Grenstein approximation is

$$P_{HG}(\Theta)=\frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}}$$

The scattering angle is

$$\cos\Theta=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi'-\varphi_0)$$

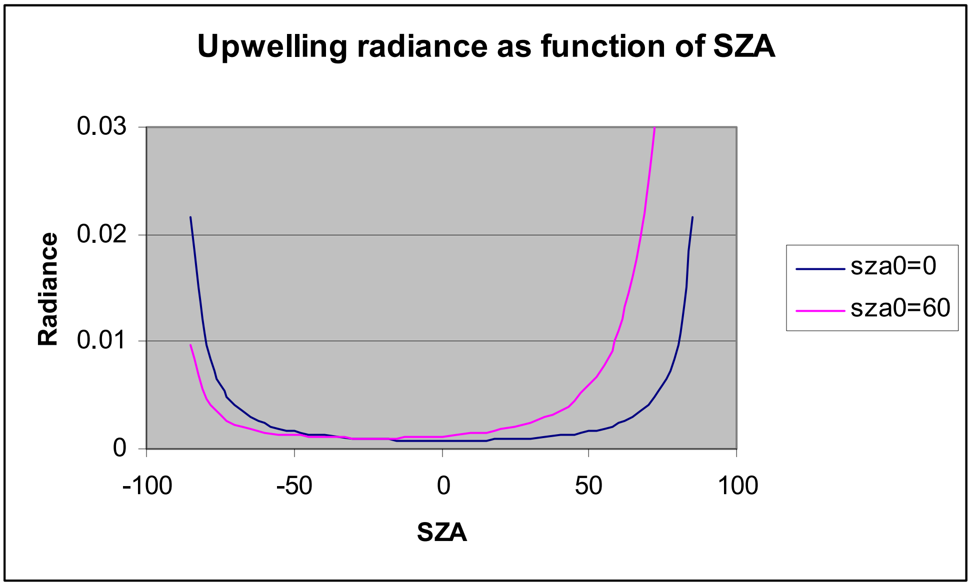
Let's consider example when scattered light propagates in zenith direction (θ=0°). Since the incident direction (θ',φ') of the direct sun flux propagation is the opposite direction from zenith, then incident zenith angle is θ'=180°-θ_0=180°-0°=180° or θ''=180°-θ_0=180°-60°=120°, and φ'=φ_0-0°=0°-0°=0°.
Therefore,

$$\cos\Theta'=\cos(0)\cos(180)+\sin(0)\sin(180)\cos(0)=-1$$

$$\cos\Theta''=\cos(0)\cos(120)+\sin(0)\sin(120)\cos(0)=1*(-0.5)+0*0.86*1=-0.5$$

Thus, the radiance scattered upward can be defined as:

$$I_\lambda^\uparrow(0,\mu,\varphi)=\frac{\omega_0F\tau_0}{4\pi\mu}P=\frac{\omega_0F\tau_0}{4\pi\mu}\frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}}$$



The upwelling radiation is inversely proportional to viewing angle (θ) and to a phase function (through a scattering angle (Θ)). The first case (*$\theta_0= 0^0$ or $\theta'=180°$*) shows that minimum of the upwelling radiation is found in the backward direction (*$\theta= 0^0$*), which is defined by SZA dependence in phase function. For the second case (*$\theta_0=6\ 0^0$ or $\theta'=120°$*), the phase function reaches maximum value at the viewing angle θ=90°. At the same time in this viewing direction we have the smallest μ. Thus, the upwelling radiation reaches maximum at θ=90°.

TASK 2

A. An aerosol layer has a visible optical depth of 0.4, single scattering albedo of 0.9 and asymmetry parameter of 0.75. Calculate the albedo for solar zenith angle of 0^0 using the Eddington and Delta-Eddington approximations for an optically thin layer. Take the forward scattering fraction $f=g^2$. Then, calculate the absorptance of the layer. Explain your results.

The albedo using Eddington approximation for the optically thin layer is expressed as following

$$R_{Edd} = \frac{\omega\tau}{\mu_0}(\frac{1}{2} - \frac{3g\mu_0}{4}) = \frac{(0.9)(0.4)}{1}(\frac{1}{2} - \frac{3(0.75)(1)}{4}) = \frac{0.36}{4}(2 - 2.25) = -0.0225$$

The delta-scaling transformation produces

$$\tau' = (1 - \omega f)\tau = (1 - 0.9 * 0.75 * 0.75) * 0.4 = 0.1975$$

$$g' = \frac{g - f}{1 - f} = \frac{g}{1 + g} = \frac{0.75}{1.75} = 0.42875$$

$$\omega' = \frac{(1 - f)\omega}{1 - \omega f} = \frac{(1 - 0.75 * 0.75) * 0.9}{(1 - 0.9 * 0.75 * 0.75)} = \frac{0.39375}{0.49375} = 0.7975$$

Therefore,

$$R_{\delta Edd} = \frac{\omega'\tau'}{\mu_0}(\frac{1}{2} - \frac{3g'\mu_0}{4}) = \frac{\omega\tau(1 - g)}{4\mu_0}(2 * (1 + g) - 3 * g * \mu_0) =$$

$$= \frac{(0.9)(0.4)(0.25)}{4 * 1}(2(1.75) - 3(0.75)(1)) = \frac{0.09}{4}(1.25) = 0.0281$$

The Eddington absorptance is the same with and without scaling:

$$A = (1-\omega')\tau' / \mu_0 = (1 - \frac{(1-f)\omega}{1-f\omega})\frac{(1-f\omega)\tau}{\mu_0} = ((1-f\omega) - \omega + f\omega)\tau / \mu_0 = (1-\omega)\tau / \mu_0$$

$$A_{Edd} = (1-0.9) * 0.4 / 1 = 0.04$$

$$A_{\delta Edd} = (1-0.7975) * 0.1975 / 1 = 0.04$$

Scaled radiative transfer system has lower optical depth and smaller single scattering, thus the same absorption.

B. Compare the results with delta-isotropic scaling.

The Delta-isotropic scattering is a more extreme case of the delta-scaling that results in the isotropic scaled asymmetry parameter *g'=0*. It usually gives less accurate results in two-stream models, but is conceptually useful because it is a one-limit of delta-scaling. For delta-isotropic scattering *f=g*, so the scaled optical parameters are

$$\tau' = (1-\omega f)\tau = (1-0.9*0.75)*0.4 = 0.13$$

$$g' = \frac{g-f}{1-f} = 0$$

$$\omega' = \frac{(1-f)\omega}{1-\omega f} = \frac{(1-0.75)*0.9}{(1-0.9*0.75)} = 0.6923$$

Thus, the delta-isotropic scattering approximation produces larger albedo as compared to the delta-Eddington approximation:

$$R_{\delta isotr} = \frac{\omega'\tau'}{\mu_0}(\frac{1}{2} - \frac{3g'\mu_0}{4}) = \frac{0.6923*0.13}{2} = 0.045$$

TASK 3

A table below shows the albedo and absorptance of a cloud layer calculated with a highly accurate multi-stream radiative transfer, Eddington and Delta-Eddington approximations. Calculate errors of approximations and show the cases for which Eddington and Delta-Eddington approximations appear to perform worse.

t	mo	r	A	r_Edd	A_Edd	r_Delta-Edd	A_Delta-Edd
1.0	1.0	0.04717	0.02387	-0.03836 (-18%)	0.02980 (24.8%)	0.04604 (-2.4%)	0.02402 (0.6%)
10	1.0	0.29783	0.31510	0.25361 (-14.8%)	0.37614 (19 %)	0.29849 (0.22%)	0.31055 (-1.44%)
1.0	0.25	0.32230	0.07568	0.36396 (12.9 %)	0.04537 (-40%)	0.27787 (-13.4 %)	0.06442 (-14.9 %)
10	0.25	0.56047	0.27518	0.56323 (0.5 %)	0.24019 (-12.7 %)	0.53892 (-3.8 %)	0.2796 (1.6 %)

Both Eddington and Delta-Eddington approximation for absorptance are worse for small optical depth (1.0) and large SZA (0.25) case. The worse Eddington approximation for albedo is found for small optical depth (1.0) and overhead sun (1.0) case (can not do well in case of strong forward scattering). The delta-Eddington produces the worse approximation of the albedo when the optical depth is small (1.0) and SZA is large (0.25). Eddington works well for approximation of isotropic-scattered radiation (large OD and large SZA). Delta-Eddington approximation works well describing forward scattering (small OD and high sun).

TASK 4

A figure below shows reflectivity (albedo), transmittance and absorptance as a function of cosine of solar zenith angle, single scattering albedo and optical depth, comparing exact, Eddington and Delta-Eddington methods.

A. Briefly describe the behavior of the albedo as a function of the cosine of solar zenith angle. Does this agree with the Eddington approximation for conservative scattering (see Fig.18.1)

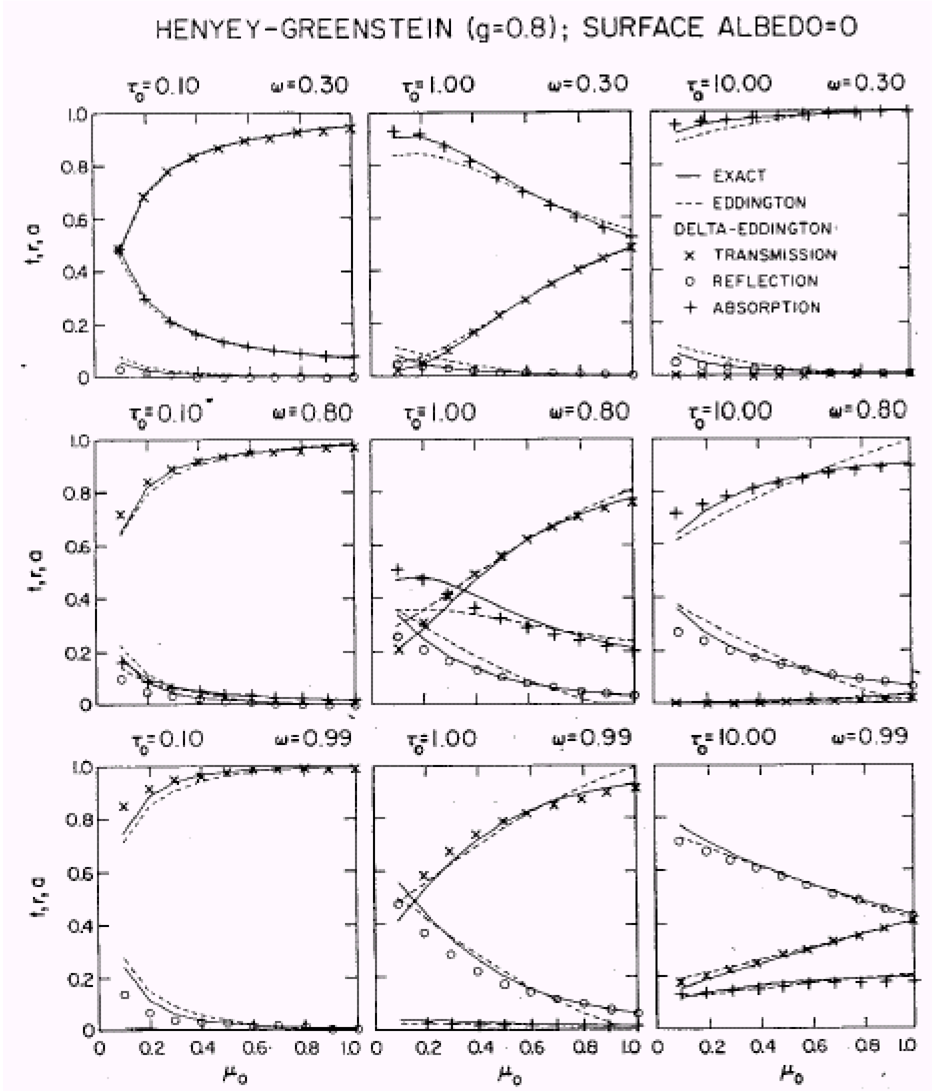
Albedo increases with SZA or reduces with cosine of SZA (large SZA results in small cosine). Eddington approximation for conservative scattering also produces albedo that increases with SZA.

B. How does aerosol absorption (i.e., single scattering albedo) affect the reflection?

Smaller single-scattering albedo (larger absoprtion) reduces reflection.

C. Compare absorptance at low and high optical depth. Explain the results.

In the optically thin atmosphere approximation (small optical depth) we saw that absorptance is inversely proportional to cosine of SZA. Thus, absorptance increases with SZA (reducing cosine). When optical depth is large, dependence of absorptance on SZA changes: absorptance slightly decreases when sun location is low (large SZA, small cosine). This is due to the fact that there is very little direct flux contribution to intensity at large SZAs, but there is large contribution from multiple-scattering.



HENYEY-GREENSTEIN (g=0.8); SURFACE ALBEDO=0
τ₀=0.10 ω=0.30
τ₀=1.00 ω=0.30
τ₀=10.00 ω=0.30
τ₀=0.10 ω=0.80
τ₀=1.00 ω=0.80
τ₀=10.00 ω=0.80
τ₀=0.10 ω=0.99
τ₀=1.00 ω=0.99
τ₀=10.00 ω=0.99
EXACT
EDDINGTON
DELTA-EDDINGTON
× TRANSMISSION
○ REFLECTION
+ ABSORPTION
t,r,a
μ₀